Algebra 3
Chapter 4 Test Review

Name Key
No Calculator Part

Tell whether each function shows growth or decay.

1. $f(x) = .5(1.25)^x$
Growth

2. $f(x) = \frac{5}{2}\left(\frac{1}{4}\right)^x$
Decay

Write each exponential expression in logarithmic form and each logarithmic expression in exponential form.

3. $3^5 = 243$
$\log_3 243 = 5$

4. $log_2 16 = 4$
$2^4 = 16$

5. $\left(\frac{1}{3}\right)^{-3} = 27$
$\log_{\frac{1}{3}} 27 = -3$

6. $2 = log 100$
$10^2 = 100$

Evaluate each.

7. $log_{12}144$
$= 2$

8. $log_{14}\frac{1}{14}$
$= -1$

9. $log\ .01$
$= -2$

10. $log_5 1$
$= 0$

11. Graph $f(x) = \left(\frac{1}{2}\right)^x$ and $f^{-1}(x)$ on the same axes. State the domain and range of each.
See Graph

Express as a single logarithm and simplify.

12. $log_2 128 - log_2 4$
$\log_2 \frac{128}{4} = \log_2 32 = 5$

13. $\log 50 + \log 2$
$\log(50 \cdot 2) = \log 100 = 2$

14. $\log 10^5 + \log 10^4$
$\log(10^5 \cdot 10^4) = \log 10^9 = 9$

15. $log_3 81^5$
$\log_3 (3^4)^5 = \log_3 3^{20} = 20$

Use the change of base formula to evaluate each.

16. $log_{125}625 = \frac{\log_5 625}{\log_5 125} = \frac{4}{3}$

17. $log_{27}9 = \frac{\log_3 9}{\log_3 27} = \frac{2}{3}$

18. $log_{64}\frac{1}{4} = \frac{\log_4 \frac{1}{4}}{\log_4 64} = \frac{-1}{3}$

Solve each equation. Check for extraneous solutions.

19. $16^x = 2^{x+2}$
$(2^4)^x = 2^{x+2}$ $x = \frac{2}{3}$

20. $27^{x-2} = 81$
$(3^3)^{x-2} = 3^4$ $x = \frac{10}{3}$

21. $log_3(x+4) = 3$
$3^3 = x+4$ $x = 23$

22. $log_5 x^2 = 2$
$5^2 = x^2$ $x = 5$

23. $log_4 100 - log_4(x+1) = 1$
$\log_4 \frac{100}{x+1} = 1$ $4^1 = \frac{100}{x+1}$ $x = 24$

24. $log_{12}x + log_{12}(x+1) = 1$
$\log_{12} x(x+1) = 1$ $12^1 = x(x+1)$ $x = -4, 3$
Extraneous

Simplify each.

25. $ln e^{3t}$
$3t$

26. $e^{\ln(x+4y)}$
$x + 4y$

27. $\ln e^5 + \ln e^2$
$= 7$

28. $e^{\ln 3} + e^{\ln 2}$
$= 5$

Describe how the graph of each is transformed from its parent function.

29. $k(x) = 4\left(-\frac{1}{2}\right)^{x-3}$
vertical stretch x4
right 3 from $f(x) = \left(-\frac{1}{2}\right)^x$

30. $m(x) = -\frac{2}{3}(x+5)^2 - 2$
reflect over x
vertical stretch x $\frac{2}{3}$
left 5
down 2

31. $a(x) = \ln(-x+4)$
reflect over y axis
left 4 from $f(x) = \ln(x)$

32. Graph the exponential function, $g(x) = -2^x + 1$. State the equation of the asymptote and the transformations of the parent function.
See Graph

Algebra 3 Name key
Chapter 4 Test Review **Calculator Part**

33. **Write the equation of the inverse of $f(x) = \frac{1}{2}x - 3$. Check your answer by graphing.**

$f^{-1}(x) = 2(x+3)$

34. **Graph $g(x) = -\ln(x+2)$. State the equation of the asymptote and the transformations of the parent function.** See graph

Solve each equation.

35. $3^{2x} = 5$

$x = \frac{\log_3 5}{2} = .73248...$

36. $10 = \ln 3^x$

$x = \frac{10}{\ln(3)} = 9.1023...$

37. $e^{x+2} = 3$

$x = \ln(3) - 2 = -.90138...$

38. **Use the change of base formula to evaluate $log_5 40$.**

$\frac{\log 40}{\log 5} = 2.2920...$

39. What is the total value of an investment of $5000 that earned 6% interest compounded continuously for 5 years?

$A = Pe^{rt}$ $A = 5000e^{(.06)5}$ $A = 6749.294$

40. A car purchased for $13,500 will depreciate in value at a rate of approximately 15% each year. Write an exponential function to model the situation. Using logarithms to solve the equation, determine how long after the purchase will the car be worth $3000.

$A = P(1-.15)^t$ $A = 13500(1-.15)^t$ $A = 13500(.85)^t$ $3000 = 13500(.85)^t$ $t = 9.2547$

41. Carbon-14 is a useful dating tool for specimens between 500 and 25,000 years old, such as ancient manuscripts and artifacts. Carabon-14's half-life is 5730 years.

a. Use $\frac{1}{2} = e^{-kt}$ to find the decay constant, k, for carbon-14.

$\frac{1}{2} = e^{-k(5730)}$ $k = .000120968...$

b. Use the natural decay function, $N(t) = N_0e^{-kt}$, to determine how much of 10 grams of carbon-14 will remain after 1000 years.

$N(1000) = 10 \cdot e^{-.000120968(1000)}$

$N(1000) = 8.8605\,g$

42. Use logarithmic regression to find a function that models the increase in the number of pepper trees in a wilderness reserve over six years. Predict the year when the number of trees will reach 70.

Year	1	2	3	4	5	6
Trees	14	30	40	46	53	55

$f(x) = 14.00549 + 23.401905 \cdot \ln(x)$

~ 10.94 years

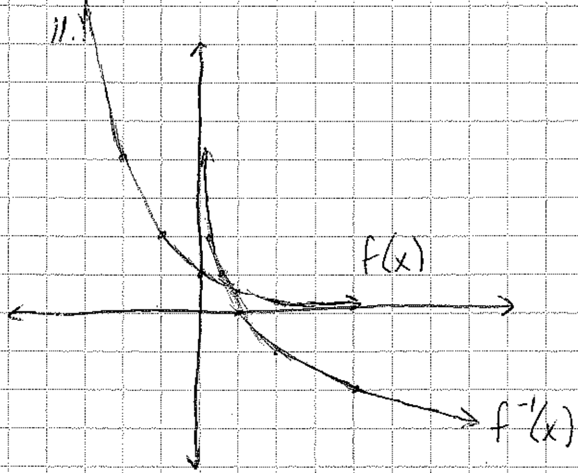
43. Use exponential regression to find a function that models the data. When will the number of telecommuters exceed 75 million?

Years After 1990	0	1	2	3	4	5	6	7	8	9	10
Telecommuters (millions)	4.4	5.5	6.6	7.3	9.1	8.5	8.7	11.1	15.7	19.6	23.6

$f(x) = 4.454 \cdot (1.16579)^x$

~ 18.406 years

11.)



$f(x) = \left(\frac{1}{2}\right)^x$ $\quad f^{-1}(x) = \log_{\frac{1}{2}} x$

$y = \left(\frac{1}{2}\right)^x \longrightarrow x = \left(\frac{1}{2}\right)^y$

$y = \log_{\frac{1}{2}} x$

$f(x)$

x	y
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

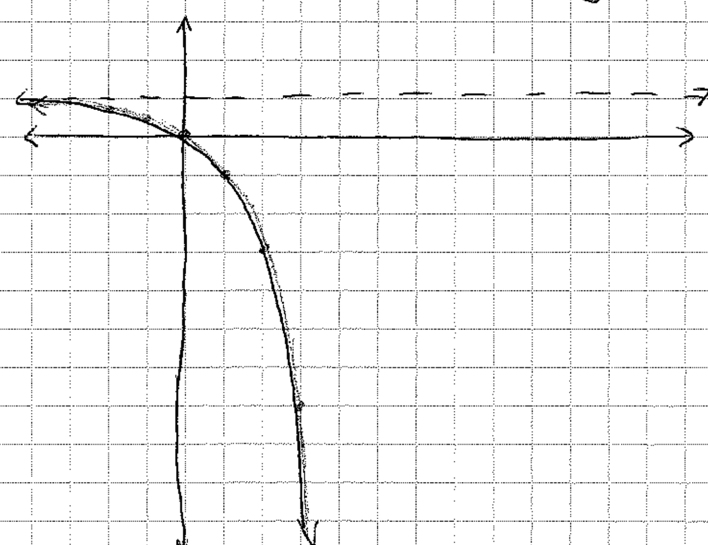
$f^{-1}(x)$

x	y
4	-2
2	-1
1	0
$\frac{1}{2}$	1
$\frac{1}{4}$	2

32.)

$g(x) = -2^x + 1$

up 1 / reflect over x axis from $g(x) = 2^x$



x	y
-2	$\frac{3}{4}$
-1	$\frac{1}{2}$
0	0
1	-1
2	-3
3	-7
4	-15

Asymptote at $y = 1$

34.) $g(x) = -\ln(x+2)$

left 2 Reflection over x axis from $f(x) = \ln(x)$ $\quad$ Asymptote at $x = -2$