Algebra 3
Quiz Review: 3.3, 3.5, 3.6

Name Key

Divide using long division.

1. $(4x^3 - 2x^2 - 6x + 1) \div (2x + 1)$

$2x^2 - 2x - 2 + \frac{3}{2x+1}$

2. $(2x^4 - 5x^2 - 6x + 7) \div (2x - 1)$

$x^3 - \frac{1}{2}x^2 - \frac{9}{4}x - \frac{15}{8} + \frac{\frac{71}{8}}{2x+1}$

Divide using synthetic division.

3. $(x^2 - 2x - 15) \div (x - 5)$

$5 \,|\; 1 \quad -2 \quad -15$
$\quad\quad\; 5 \quad 15$
$\quad 1 \quad 3 \quad 0$

$x + 3$

4. $(x^3 - 3x^2 + x - 3) \div (x + 1)$

$-1 \,|\; 1 \quad -3 \quad 1 \quad -3$
$\quad\quad -1 \quad 4 \quad -5$
$\quad 1 \quad -4 \quad 5 \quad -8$

$x^2 - 4x + 5 + \frac{8}{x+1}$

Use synthetic substitution (use the value provided with synthetic division and find the remainder) to evaluate each polynomial for the given value.

5. $f(x) = 8x^4 - 4x^2 + x + 4; x = -\frac{1}{2}$

$-\frac{1}{2} \,|\; 8 \quad 0 \quad -4 \quad 1 \quad 4$
$\quad\quad -4 \quad 2 \quad 1 \quad -1$
$\quad 8 \quad -4 \quad -2 \quad 2 \quad \boxed{3}$

$f(-\frac{1}{2}) = 3$

Determine whether the given binomial is a factor of *P(x)*.

6. $P(x) = 4x^3 - 2x^2 - x + 1; (x + 4)$

$-4 \,|\; 4 \quad -2 \quad -1 \quad 1$
$\quad\quad -16 \quad +72 \quad -284$
$\quad 4 \quad -18 \quad 71 \quad -283$

No

7. $P(x) = x^4 - 2x^3 + x^2 - 3x + 2; (x - 2)$

$2 \,|\; 1 \quad -2 \quad 1 \quad -3 \quad 2$
$\quad\quad 2 \quad 0 \quad 2 \quad -2$
$\quad 1 \quad 0 \quad 1 \quad -1 \quad 0$

yes

Use the Fundamental Theorem of Algebra to determine how many roots the polynomial has?

8. $P(x) = x^5 - 6x^4 + 4x^3 + 17x^2 - 5x - 6$

5 Roots

9. $P(x) = 2x^4 - 4x^3 - 2x^2 + 3x - 5$

4 roots

Use the Rational Roots Theorem to identify <u>all of the possible rational roots</u> of the polynomial.

10. $P(x) = x^5 + 4x^4 - x^3 - 9x^2 + 6$

$\frac{\pm\{1, 2, 3, 6\}}{\pm\{1\}} = \pm 1, \pm 2, \pm 3, \pm 6$

11. $P(x) = 2x^3 - 5x^2 - 28x + 15$

$\frac{\pm\{1, 3, 5, 15\}}{\{1, 2\}} = \pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$

(Solve) each of the polynomial equations by factoring or using the reverse binomial theorem. State the multiplicity of each root when applicable.

12. $x^4 - 16 = 0$ $x = 2, -2, 2i, -2i$

$(x^2+4)(x^2-4) = 0$

$(x+2i)(x-2i)(x+2)(x-2) = 0$

13. $x^4 - 11x^2 + 28 = 0$ $x = 2, -2, \sqrt{7}, -\sqrt{7}$

$(x^2-4)(x^2-7) = 0$

$(x+2)(x-2)(x+\sqrt{7})(x-\sqrt{7}) = 0$

14. $3x^5 - 3x^3 - 60x = 0$ $x = 0, 2i, -2i, \sqrt{7}, -\sqrt{7}$

$3x(x^4 - x^2 - 20) = 0$

$3x(x^2+4)(x^2-5) = 0$

$3x(x+2i)(x-2i)(x+\sqrt{5})(x-\sqrt{5}) = 0$

15. $27x^3 + 135x^2 + 225x + 125 = 0$

$(3x+5)^3 = 0$

$3x + 5 = 0$ $x = \frac{-5}{3}$

Given the indicated roots of each polynomial, use the Irrational and Complex Conjugate Root Theorems to determine the remaining zeros.

16. $P(x)$ is a polynomial of degree four with zeros 2, 3 and $2i$ $-2i$

17. $P(x)$ is a polynomial of degree five with zeros -5, $-5 + 7i$, and $3 + \sqrt{2}$

$-5-7i$ & $3-\sqrt{2}$

Identify all of the roots (real and complex) of each equation.

18. $2x^3 + 7x^2 - 53x - 28 = 0$

$(x-4)(x+7)(2x+1) = 0$

$x = 4, -7, \frac{-1}{2}$

19. $2x^3 + 17x^2 + 23x - 42 = 0$

$(x+6)(x-1)(2x+7) = 0$

$x = -6, 1, \frac{-7}{2}$

20. $x^4 + 4x^3 + 13x^2 + 36x + 36 = 0$

$(x+2)(x+2)(x^2+9) = 0$

$x^2 = -9$ $x = \pm 3i$

$x = -2, +3i, -3i$

21. $2x^4 - 13x^3 + 23x^2 - 52x + 60 = 0$

$(x-5)(2x-3)(x^2+4) = 0$

$x = 5, \frac{3}{2}, 2i, -2i$

Write a polynomial with the given zeros.

22. 4, $\frac{1}{2}$, and $-3i$ and $3i$

$f(x) = (x-4)(x-\frac{1}{2})(x+3i)(x-3i)$

$f(x) = (x^2 - \frac{9}{2}x + 2) \cdot (x^2+9)$

$= x^4 + 11x^2 - \frac{9}{2}x^3 - \frac{81}{2}x + 18$

or $= 2x^4 + 22x^2 - 9x^3 - 81x + 36$ would work as well

23. $\sqrt{2}, -7,$ and 0 and $-\sqrt{2}$

$(x-\sqrt{2})(x+\sqrt{2})(x+7)(x+0)$

$(x^2-2)(x+7) \cdot x$

$x^4 + 7x^3 - 2x^2 - 14x$