Matrix Test Review
Algebra 3

Name key
Date _____ Block _____

For #1-3, state the dimensions of the matrix.

1. $\begin{bmatrix} 1 & 4 & -6 \\ 2 & -3 & -3 \end{bmatrix}$ 2×3

2. $\begin{bmatrix} 9 \\ -8 \\ 2 \end{bmatrix}$ 3×1

3. $\begin{bmatrix} -2 & 3 & -1 \\ 0 & 0 & 2 \\ -1 & 8 & 2 \end{bmatrix}$ 3×3

For #4-6, determine if each matrix product is possible. If so, state the dimensions of the product.

4. $A_{1\times5} \cdot B_{5\times7}$ yes, 1×7

5. $A_{2\times5} \cdot B_{2\times5}$ No

6. $A_{3\times2} \cdot B_{2\times4}$ yes, 3×4

For #7-9, determine the dimensions of matrix M.

7. $A_{1\times3} \cdot M = B_{1\times4}$ $M_{3\times4}$

8. $A_{2\times4} \cdot M = B_{2\times1}$ $M_{4\times1}$

9. $A_{3\times3} \cdot M = B_{3\times5}$ $M_{3\times5}$

For #10 -, use matrices A-H.

$A = \begin{bmatrix} 1 & 4 & -6 \\ 2 & -3 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 0 & 6 \\ 4 & -3 & 8 \end{bmatrix}$ $C = \begin{bmatrix} 5 & 2 \\ 2 & -1 \\ 5 & 4 \end{bmatrix}$ $D = \begin{bmatrix} -2 & 2 \\ 0 & -1 \\ -5 & 7 \end{bmatrix}$ $E = \begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}$

$F = \begin{bmatrix} -2 & 3 & -1 \\ 0 & 0 & 2 \\ -1 & 8 & 2 \end{bmatrix}$ $G = \begin{bmatrix} 5 & 3 & 0 \\ 0 & 4 & -1 \\ 3 & -5 & 0 \end{bmatrix}$ $H = \begin{bmatrix} 9 \\ -8 \\ 2 \end{bmatrix}$

10. $2A - \frac{1}{3}B$

$2\begin{bmatrix} 1 & 4 & -6 \\ 2 & -3 & -3 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} 3 & 0 & 6 \\ 4 & -3 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 8 & -14 \\ \frac{8}{3} & -5 & -\frac{26}{3} \end{bmatrix}$

11. $C - D$

$\begin{bmatrix} 5 & 2 \\ 2 & -1 \\ 5 & 4 \end{bmatrix} - \begin{bmatrix} -2 & 2 \\ 0 & -1 \\ -5 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 2 & 0 \\ 10 & -3 \end{bmatrix}$

12. $D \cdot A$

$\begin{bmatrix} -2 & 2 \\ 0 & -1 \\ -5 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 & -6 \\ 2 & -3 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -14 & 6 \\ -2 & 3 & 3 \\ 9 & -41 & 9 \end{bmatrix}$

3×2 · 2×3 = 3×3

13. E^{-1} (By hand)

$E^{-1} = \begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}^{-1} = \frac{1}{14}\begin{bmatrix} 1 & -4 \\ 3 & 2 \end{bmatrix}$

14. F^{-1} (By calculator)

$$\begin{bmatrix} -2 & 3 & -1 \\ 0 & 0 & 2 \\ -1 & 8 & 2 \end{bmatrix}^{-1} = \frac{1}{26}\begin{bmatrix} -16 & 14 & 6 \\ -2 & -5 & 4 \\ 0 & 13 & 0 \end{bmatrix}$$

$\det = 26$

15. $-2F + G$

$$-2\begin{bmatrix} -2 & 3 & -1 \\ 0 & 0 & 2 \\ -1 & 8 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 3 & 0 \\ 0 & 4 & -1 \\ 3 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 9 & -3 & 2 \\ 0 & 4 & -5 \\ 5 & -21 & -4 \end{bmatrix}$$

16. $C \cdot D$

$$\underset{3\times 2}{\begin{bmatrix} 5 & 2 \\ 2 & -1 \\ 5 & 4 \end{bmatrix}} \cdot \underset{3\times 2}{\begin{bmatrix} -2 & 2 \\ 0 & -1 \\ -5 & 7 \end{bmatrix}} = \text{Not possible}$$

17. $G \cdot H$

$$\begin{bmatrix} 5 & 3 & 0 \\ 0 & 4 & -1 \\ 3 & -5 & 0 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ -8 \\ 2 \end{bmatrix} = \begin{bmatrix} 21 \\ -34 \\ 67 \end{bmatrix}$$

18. $B \cdot D$

$$\underset{2\times 3}{\begin{bmatrix} 3 & 0 & 6 \\ 4 & -3 & 8 \end{bmatrix}} \cdot \underset{3\times 2}{\begin{bmatrix} -2 & 2 \\ 0 & -1 \\ -5 & 7 \end{bmatrix}} = \begin{bmatrix} -36 & 48 \\ -48 & 67 \end{bmatrix}$$

19. Determinant of F

$\det(F) = 26$

20. Determinant of E

$\det(E) = 14$

21. Find AB

$$A = \underset{3\times 2}{\begin{bmatrix} 5 & 6 \\ 2 & 3 \\ -4 & 0 \end{bmatrix}} \quad B = \underset{2\times 3}{\begin{bmatrix} 8 & 0 & -3 \\ 2 & 6 & 4 \end{bmatrix}} = \begin{bmatrix} 52 & 36 & 9 \\ 22 & 18 & 6 \\ -32 & 0 & 12 \end{bmatrix}$$

22. Find the determinant by hand of $A = \begin{bmatrix} 5 & -3 & 2 \\ 6 & 10 & -1 \\ 1 & -2 & 0 \end{bmatrix}$

$$= 5\begin{vmatrix} 10 & -1 \\ -2 & 0 \end{vmatrix} - (-3)\begin{vmatrix} 6 & -1 \\ 1 & 0 \end{vmatrix} + 2\begin{vmatrix} 6 & 10 \\ 1 & -2 \end{vmatrix}$$

$$= 5(0-2) + 3(0+1) + 2(-12-10)$$

$$= -10 + 3 + -44$$

$$= \boxed{-51}$$

For #23 – 26, solve the systems using matrices. Show the matrix equation you used.

23. $\begin{cases} -3x - 4y = 9 \\ 9x + 10y = -3 \end{cases}$

$\begin{bmatrix} -3 & -4 \\ 9 & 10 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -3 \end{bmatrix}$ $[X] = [A]^{-1} \cdot [B]$

$[A] \cdot [X] = [B]$ $[X] = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ -12 \end{bmatrix}$

24. $\begin{cases} x + 2y = 2 \\ 3x + 6y = 6 \end{cases}$

No Solution

25. $\begin{cases} 2x - y + 2z = 12 \\ x + 2y - 2z = -11 \\ 2x + y + 3z = 12 \end{cases}$

$\begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -11 \\ 12 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$

26. $\begin{cases} -2x - 4z = 2 \\ -3y + 5z = -14 \\ x + 2y = 7 \end{cases}$

$\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$

27. Give an example of 2 matrices that can be multiplied. Why?

$\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}_{3\times 2} \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}_{2\times 2} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix}_{3\times 2}$ Columns of first are same as rows of second

28. Give an example of 2 Matrices that CAN NOT be multiplied. Why?

$\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}_{3\times 2} \cdot \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}_{3\times 2}$ = Not possible Columns of first do not match rows of second

Applications Review Matrices

1.) A store owner can buy a second hand vending machine for $120. If a bottle of pop can be bought for $.65 and sold for $.95, how many bottles must be sold to recover the costs incurred? Note; this is called the break-even point (where expenses exactly equals income). It is used to help businesses estimate the amount of a product that must be sold before a profit can be made

2.) The local electric company's current rate schedule is $8.50 equipment charge and $.09 per Kwh of electricity used. Next year the company plans to charge a base fee of $6.00 and a $.10 per Kwh electricity charge. For what Kwh use-age will a home-owner's bill be the same for both years?

3.) A farmer has 1200 hectares planted in potatoes, grain and corn. When the crops are planted, each hectare of potatoes requires 3 hours to plant, grain requires 1 hour and corn requires 2 hours for each hectare. Labour costs $12 per hour. Seed costs are $10 per hectare for potatoes, $15 per hectare for grain and $5 per hectare for corn. The farmer has planned to spend $24,000 on labour and $13,000 on seed. How many hectares of each crop can be planted?

p = # H. of potatoes
g = # H. of grain
c = # H of corn

$p+g+c = 1200$
$12(3p+1g+2c) = 24000$
$10p+15g+5c = 13000$

$p = 200$
$g = 600$
$c = 400$

4.) Using what you know about matrix coding and decoding, encode the following messages using the encoding matrix:

I used 0 for spaces in the following Examples.

$$A=\begin{bmatrix}4 & -2\\ 3 & -2\end{bmatrix}$$

1. THIS MAN CANNOT BE TRUSTED 104, -56, 93, -56, 39, -26, 46, -30, 9, -6, 46, -30, 101, -58, 80, -40, 23, -14, 60, -40

2. YOU ARE BEING FOLLOWED 145, -80, 84, -42, 58, -38, 20, -18, 23, -14, 78, -46, 28, -14, 69, -42, 84, -48, 129, -76, 32, -18, 135, -78, 136, -78, 32, -18

3. MEET ME AT THE WHITE HOUSE
67, -36, 80, -50, 39, 26, 20, -10, 64, -42, 60, -40, 47, -26, 69, -46, 59, -34, 95, -50, 24, -16, 123, -72, 91, -48

4. THIS PHONE IS NOT SECURE
104, -56, 93, -56, 48, -32, 77, -46, 71, -38, 27, -18, 76, -38, 101, -58, 80, -40, 91, -48, 75, -48, 87, -46

Using what you know about encoding and decoding matrices, DECODE the following using the original encoding matrix (a 0 will correspond to a space):

$$A=\begin{bmatrix}4 & -2\\ 3 & -2\end{bmatrix}$$

Message 1	104	-56	93	-56	27	-18	76	-38	91	-48	66	-42	80	-50	0	0
							THISISSECRET									
Message 2	77	-46	92	-46	15	-8	56	-28	107	-56	69	-46	78	-46	0	0
							HOW CAN WE WIN									